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International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Monday 15 November 2010 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

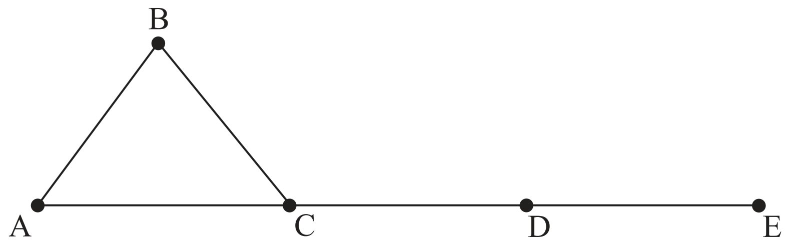
- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

4 pages

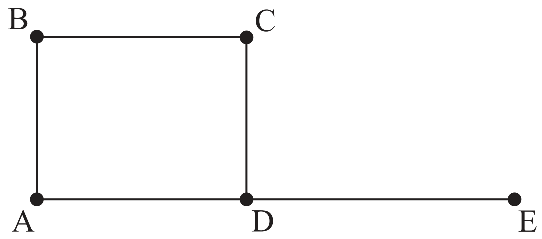
Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 12]*

(a) (i) Write down the degree of each vertex for the following two graphs.



Graph *G*



Graph *H*

(ii) Are graphs G and H isomorphic? Justify your answer. *[4 marks]*

(b) (i) A graph is simple, planar and connected. Write down the inequality connecting v and e, and give the condition on v for this inequality to hold.

(ii) Sketch a simple, connected, planar graph with $v = 2$ where the inequality from part (b)(i) is not true.

(iii) Sketch a simple, connected, planar graph with $v = 1$ where the inequality from part (b)(i) is not true.

(iv) Given a connected, planar graph with v vertices, v^2 edges and 8 faces, find v. Sketch a graph that fulfils all of these conditions. *[8 marks]*

2. *[Maximum mark: 11]*

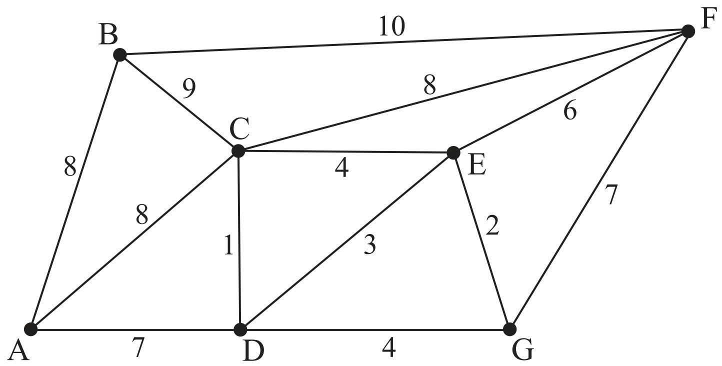
(a) Find the general solution for the following system of congruences.

$$N \equiv 3 \pmod{11}$$
$$N \equiv 4 \pmod{9}$$
$$N \equiv 0 \pmod{7}$$ *[9 marks]*

(b) Find all values of N such that $2000 \le N \le 4000$. *[2 marks]*

3. *[Maximum mark: 12]*

Consider the following weighted graph.



(a) (i) Use Kruskal's algorithm to find the minimum spanning tree. Indicate the order in which you select the edges and draw the final spanning tree.

(ii) Write down the total weight of this minimum spanning tree. *[8 marks]*

(b) Sketch a spanning tree of maximum total weight and write down its weight. *[4 marks]*

4. *[Maximum mark: 11]*

(a) Write down Fermat's little theorem. *[2 marks]*

(b) In base 5 the representation of a natural number X is $\left(k00013(5-k)\right)_5$. This means that $X = k\times 5^6 + 1\times 5^2 + 3\times 5 + (5-k)$.

In base 7 the representation of X is $(a_n a_{n-1} \ldots a_2 a_1 a_0)_7$.

Find a_0. *[5 marks]*

(c) Given that $k = 2$, find X in base 7. *[4 marks]*

5. *[Maximum mark: 14]*

(a) A graph has n vertices with degrees 1, 2, 3, …, n. Prove that $n \equiv 0 \pmod 4$ or $n \equiv 3 \pmod 4$. *[6 marks]*

(b) Let G be a simple graph with n vertices, $n \geq 2$. Prove, by contradiction, that at least two of the vertices of G must have the same degree. *[8 marks]*
